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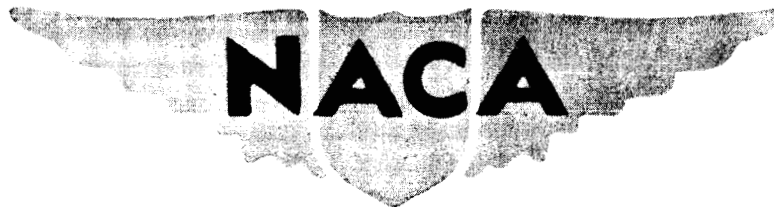
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A PROCEDURE FOR THE SHEAR-LAG ANALYSIS OF BOX BEAMS

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ADVANCE ██████ REPORT

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SUMMARY

A procedure for the shear-lag analysis of box beams, such as wing structures, is outlined. A previously published method separated the most essential part of the shear-lag analysis from the Mc/I analysis. The present method, by slightly modifying the computation of the chordwise distribution of stress, entirely separates the shear-lag analysis from the Mc/I analysis. The discussion points out that savings in time and greater accuracy will result from this separation.

INTRODUCTION

A comprehensive paper on shear lag (reference 1) published previously gives two methods of analysis. The method recommended for ordinary use achieves some of its simplicity and speed by separating the main part of the shear-lag analysis from the Mc/I analysis; the two analyses are combined just before the chordwise distribution of the stringer stresses is computed. When the computation of the individual stringer stresses is considered necessary, the method recommended in reference 1 should be slightly modified to keep the shear-lag analysis separate from the Mc/I analysis until the final addition of stresses is made. The present paper gives the complete outline of the modified procedure and discusses the advantages gained by separating the shear-lag analysis from the Mc/I analysis. The treatment is chiefly intended for wing structures of the central-box type.

THE SHEAR-LAG ANALYSIS OF BOX BEAMS

General Procedure of Analysis

The general procedure for shear-lag analysis consists of the following steps:

1. At a number of stations along the span, preliminary section moduli are calculated; estimated amounts of skin are assumed to work with the stringers on the compression side. Preliminary values of the stringer stresses are calculated with these moduli.

2. The preliminary stringer stresses are used to calculate the effective widths of skin. Marguerre's formula for the effective width

$$2w/b = \sqrt[3]{\sigma_{cr}/\sigma} \qquad (1)$$

is recommended as a good compromise between accuracy and simplicity. Average values of the critical stress σ_{cr} of the sheet and of the stringer stress σ may be used for any station unless the thickness of the skin varies chordwise.

3. After the effective widths of sheet have been established by checking them for agreement with the assumed values, final section moduli are calculated and the final Mc/I analysis is made.

4. The shear-lag analysis is made by the methods to be described later; the result of this analysis is a set of stress corrections.

5. The stress corrections resulting from the shear-lag analysis are added to the stresses resulting from the final Mc/I analysis.

It should be noted that steps 1 to 3 constitute the Mc/I analysis familiar to every analyst, with the exception, perhaps, of formula (1) for the effective width. There are no restrictions placed on the Mc/I analysis; lift and drag forces may be considered separately or jointly and principal axes may or may not be used, as desired.

Basic Procedure for Shear-Lag Analysis

The basic procedure for shear-lag analysis is the procedure applicable to box beams of rectangular cross section symmetrical about a spanwise plane parallel to the plane of the loading being analyzed. As a rule, only beam loads need be investigated; the shear-lag effect on chord

loads may be neglected. Lift loads may often be used in place of beam loads. Symmetry being assumed to exist, all calculations are made for the half-section (fig. 1).

The individual steps of the basic procedure are as follows:

1. The beam is divided into a number of bays, so that the properties of the cross section and the applied running shear may be considered as sensibly constant within each bay. The stations are numbered as shown in figure 2.

2. For each bay, the following properties are computed:

A_F area of corner flange. (This area consists chiefly of two items: the area of the corner angles when they exist, and an area $1/6\ ht_w$ that takes into account the portion of the bending carried by the shear webs.)

A_L area of all longitudinals (stringers and effective widths of skin)

$A_T = A_F + A_L$

b.   width of half-section

b_r width at root

t   thickness of cover sheet (chordwise average if there is a variation)

t_f fictitious thickness of cover sheet $(t\ b_r/b)$

G   effective shear modulus of cover sheet   (This modulus should be estimated on the basis of the combined stresses acting on the sheet at the load being investigated — for instance, limit load or ultimate design load.)

K   shear-lag parameter $\left[K^2 = \dfrac{2Gt}{Eb}\left(\dfrac{1}{A_F} + \dfrac{1}{A_L}\right) \right]$ (2)

L   length of bay

$p = \dfrac{K}{Gt_f \tanh KL}$ (3)

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$$q = \frac{K}{Gt_f \sinh KL} \tag{4}$$

$$\gamma = \frac{S_W A L}{Gt_f h A_T} \tag{5}$$

where S_W is the shear force in the shear web; that is, the external shear force S_E reduced by the vertical component of the stringer forces if the beam tapers in depth

$$S_W = S_E - \frac{M}{h} \tan \delta \tag{6}$$

For numerical work, it will be found convenient to modify the expressions for p, q, and γ by writing G in terms of a reference modulus — for example, the shear modulus of the material. The value of the reference modulus may then be omitted from the expressions for p, q, and γ, as will be seen from an inspection of the next step. The thickness t may be treated in the same manner.

3. A set of equations is written for the statically indeterminate forces X, in which the coefficients p, q, and γ are used,

$$X_0 q_1 - X_1 (p_1 + p_2) + X_2 q_2 = -\gamma_1 + \gamma_2$$

$$X_1 q_2 - X_2 (p_2 + p_3) + X_3 q_3 = -\gamma_2 + \gamma_3$$

$$\cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots$$

$$X_{n-1} q_n - X_n (p_n + p_{n+1}) + X_{n+1} q_{n+1} = -\gamma_n + \gamma_{n+1} \tag{7}$$

$$\cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots$$

$$X_{r-1} q_r - X_r (p_r + p_{r+1}) = -\gamma_r + \gamma_{r+1}$$

4. Some of the quantities appearing in the set of equations are determined by the boundary conditions as follows:

    (a) If station zero is at the extreme tip, $X_0 = 0$.

    (b) If station zero is at the joint between the end of the box beam and a special tip structure connected in such a way that only the corner flanges carry stress across the joint,

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$$X_0 = \frac{MA_L}{hA_T} \tag{8}$$

(c) If the longitudinals stop at the root section and do not carry stress into the fuselage, the last equation of the system drops out and

$$X_r = \frac{MA_L}{hA_T} \tag{9}$$

In formulas (8) and (9), the terms on the right-hand side are, of course, those appropriate to the section under consideration.

(d) If the longitudinals do not stop at the root section; that is, if the wing cover being analyzed is continuous through the fuselage,

$$\gamma_{r+1} = 0$$

because $S_W = 0$ in the fuselage and

$$p_{r+1} = \frac{2L}{bE}\left(\frac{1}{A_F} + \frac{1}{A_L}\right) \tag{10}$$

5. The set of equations is solved and the values of X are tabulated.

6. The correction to the Mc/I stress in the corner flange at a given station is

$$\Delta\sigma_F = \frac{X}{A_F} \tag{11}$$

The correction to the Mc/I stress in a stringer located at a distance y from the center line of the beam is

$$\Delta\sigma = \Delta\sigma_F - D\left[1 - \left(\frac{y}{b}\right)^3\right] \tag{12}$$

where

$$D = \frac{4}{3} X\left(\frac{1}{A_F} + \frac{1}{A_L}\right) \tag{13}$$

The addition of the corrections given by formulas (11) and (12) to the Mc/I stresses completes the analysis of the stringer stresses (fig. 3). The shear stresses in the sheet may then be obtained by statics from the equilibrium of longitudinal forces on sections of the cover.

For a single concentrated load applied to a beam of constant cross section, figure 4 shows the Mc/I stresses in the flange, the shear-lag corrections $\Delta\sigma_F = X/A_F$, and the total flange stresses. It will be noted that the sign of the correction changes along the span and that there is one station at which there is no correction to the Mc/I stress. Similar curves apply to beams with distributed loading. The station with zero correction is of practical interest when a strain survey is to be made at a single station, because strain readings taken too close to this station would be misleading.

The formula (12) is an approximation that is more convenient for practical use than the theoretical expression involving hyperbolic functions (reference 1). The approximation breaks down to some extent near stations where the longitudinals stop. In such regions, the stringer stresses near the center line approach zero, while the stresses given by the approximation (12) pass through zero and change sign. No practical significance attaches to this failure of the approximation, because it occurs only where the stresses are too small to be of interest.

Modifications of Basic Procedure

Modifications for camber of cover.- Only slight modifications are necessary to adapt the procedure to the analysis of beams with slightly cambered covers. The expression (2) is replaced by

$$K^2 = \frac{2Gt}{Eb'}\left(\frac{1 + \dfrac{c_1+c_2}{h_W}}{A_F} + \frac{1}{A_L}\right) \tag{14}$$

where b' is the developed half-width of the cover sheet and c_1 and c_2 are the effective cambers, that is, the vertical distances from the corner flanges to the centroids of the stringer forces. The stresses being unknown at the outset, the locations of the force centroids must be estimated; for most practical purposes, the centroids of the

stringer areas A_L may be used (fig. 5). The formulas
(5) and (6) are replaced by the more general expressions

$$\gamma = \frac{S_W A_L z_L}{G t_f I} \qquad (15)$$

$$S_W = S_E - \frac{MQ}{I} \tan \delta \qquad (16)$$

where z is the distance from the neutral axis and Q is
the static moment of A_L and A_F about the neutral axis.
From this point on, the procedure for calculating the X-
forces is identical with the basic procedure.

The correction to the Mc/I stress of the corner
flange is

$$\Delta\sigma_F = \frac{X}{A_F} \left(1 + \frac{c}{h_W} \right) \qquad (17)$$

where c is the effective camber of the cover being anal-
yzed. The correction to the Mc/I stress in a stringer
located at a distance y from the center line of the beam
is given by the expression

$$\Delta\sigma = \frac{X}{A_F} - D \left[1 - \left(\frac{y}{b} \right)^3 \right] \qquad (18)$$

where D is given by formula (13).

The shear-lag effect decreases the stringer stresses
near the center line of the beam and increases them near
the corner flange. This shift decreases the effective
beam depth when the cover is cambered; a correction must
therefore be applied not only to the side being analyzed —
for example, the compression side — but also to the oppo-
site side, the tension side. The magnitude of the correc-
tion force is Xc/h_W. This force increases the flange
stress on the opposite side when X is positive.

Modifications for unsymmetrical cross sections.— No
satisfactory theoretical treatment of shear-lag effect in
beams with unsymmetrical cross sections appears to have
been published so far. There is in the literature a paper
dealing with the analysis of beams of unsymmetrical closed

section and a paper dealing with beams of unsymmetrical open cross section. Both papers contain several assumptions of questionable validity. The second paper, moreover, omits the vertical component of the shear in the cover sheet; at best, then, the formulas given in this paper would be approximations valid only when the dissymmetry is very small.

The lack of a satisfactory theory makes it impossible at present to give a rational procedure for beams with very unsymmetrical cross sections. Fortunately, the dissymmetry of practical cross sections is seldom very marked, and it is permissible to resort to the common expedient of using a mean cross section obtained by averaging the values of h_W and A_F for the front and rear spars. It should be carefully noted that <u>the mean cross section is used only to calculate the shear-lag corrections, not the Mc/I stresses.</u> The error committed by using a mean section then affects only the shear-lag corrections $\Delta\sigma$ and is, therefore, a very small fraction of the total stress, the shear-lag correction itself being only a fraction of the total stress.

<u>Analysis of beams with cut-outs.</u>- The analysis of beams with small or medium cut-outs is effected conveniently by the method of liquidating forces (reference 1). The beam is first analyzed on the assumption that no cut-out has been made. The internal forces at the boundaries of the proposed cut-out are next calculated. Liquidating forces are then introduced, equal and opposite to the boundary forces just found. The stresses caused by the liquidating forces are calculated and are superposed on the stresses calculated for the box without cut-out. The details of the calculation for certain types of cut-outs may be found in references 2 and 3.

Beams with full-width, or nearly full-width, cut-outs are analyzed in two parts. The part outboard of the cut-out is analyzed as a beam with longitudinals interrupted at the "root," that is, at the outboard end of the cut-out. The part inboard of the cut-out is analyzed as a beam with a wing tip structure introducing moments at the tip, that is, at the inboard end of the cut-out.

Preliminary Estimates of Shear-Lag Effects

The general procedure for analysis is very flexible and can be easily adapted to the requirements of preliminary

analysis or first estimates. The Mc/I analysis, as well as the shear-lag analysis, can be simplified by substituting a mean surface without camber for a cambered surface. The shear-lag analysis can be simplified by reducing the number of bays into which the beam is divided.

For a beam with longitudinals continuous through the fuselage, a first estimate of the shear-lag effect may be made under the assumptions that the carry-through members are rigid, that the load is uniformly distributed along the span, and that the cross section is constant. The theoretical formulas for this case give for the ratio of the actual flange stress at the root to the Mc/I stress

$$R_F = 1 + \frac{2A_L}{A_F KL} \tag{19}$$

and for the ratio of the average stringer stress (not the stringer stress at the center line) to the Mc/I stress

$$R_L = 1 - \frac{2}{KL} \tag{20}$$

where L is the semispan. When a concentrated load is applied at the tip, the factor 2 appearing in formulas (19) and (20) must be omitted. This difference must be borne in mind when the formulas are applied to test beams loaded at the tip or when empirical coefficients derived from such test beams are applied to wings.

DISCUSSION

The most important feature of the suggested method of analysis is the complete separation of the shear-lag analysis from the Mc/I analysis. In contrast with the methods that do not effect such a separation, the following advantages are gained:

1. The simplifying assumptions that are necessary at the present state of knowledge (for instance, the assumption of symmetry of the cross section) cause errors in only the stress corrections $\Delta\sigma$, not in the Mc/I stresses. The stress corrections being small compared with the Mc/I stresses, the errors will be very small compared with the total stresses.

2. The time required is reduced by breaking the analysis down into independent operations (Mc/I analysis and shear-lag analysis). This breakdown makes it possible to employ more men simultaneously, in accordance with a recognized principle of production.

3. The breakdown into independent operations makes it possible to use each man more efficiently. The engineer capable of dealing with shear-lag analyses is not required to make routine Mc/I analyses.

4. The method is very flexible and can be easily adapted to rough estimates, preliminary analyses, or final analyses by varying the number of the bays for the shear-lag analyses and by using well-known short-cuts for the Mc/I analyses.

Other features of the method that should be taken into account when making comparisons with other methods are the following:

No empirical coefficients are used. This point is important because shear lag is affected by so many variables that it is very difficult to deduce reliable empirical coefficients from tests. Factors that are often completely neglected — for instance, the properties of the carry-through members in the fuselage — may decisively influence the stresses in a given case. Empirical coefficients obtained by neglecting such factors may be very misleading.

The difficulty of obtaining reliable experimental data is accentuated by the fact that structures built up from sheet metal show considerable irregularities in behavior. Extensive strain surveys on a large number of specimens should therefore be made before conclusions are drawn about the reliability of a proposed method of analysis. A small number of strain measurements on a single wing might be acceptable for proving the validity of a theory that does not contain any questionable simplifying assumption; the more questionable the simplifying assumptions are and the more the method relies on empirical coefficients, the more necessary it becomes to extend the range of the experimental proofs. On the basis of experimental proofs submitted, the method given here scores over any other published method.

Only mathematical processes with which the average analyst is entirely familiar are employed. The most dif-

ficult step is the solution of a system of equations analogous to the ordinary three-moment equations. The average analyst is acquainted with such equations; he is not well acquainted, on the other hand, with Fourier series or with the solution of differential equations.

In conclusion, it may be pointed out that every step of the analysis is a calculation performed in a definite manner. The analyst is neither expected nor required to substitute engineering judgment for calculation. Engineering judgment is indispensable in design and can be effectively used to speed up preliminary analysis. For obvious reasons, however, a method of final analysis should be as free as possible of personal factors.

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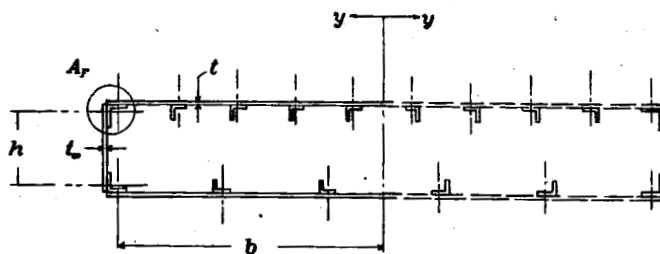
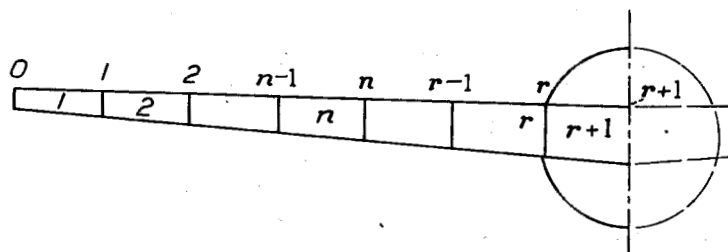Figure 1.- Cross section of box beam.
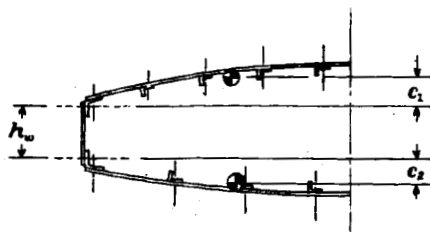
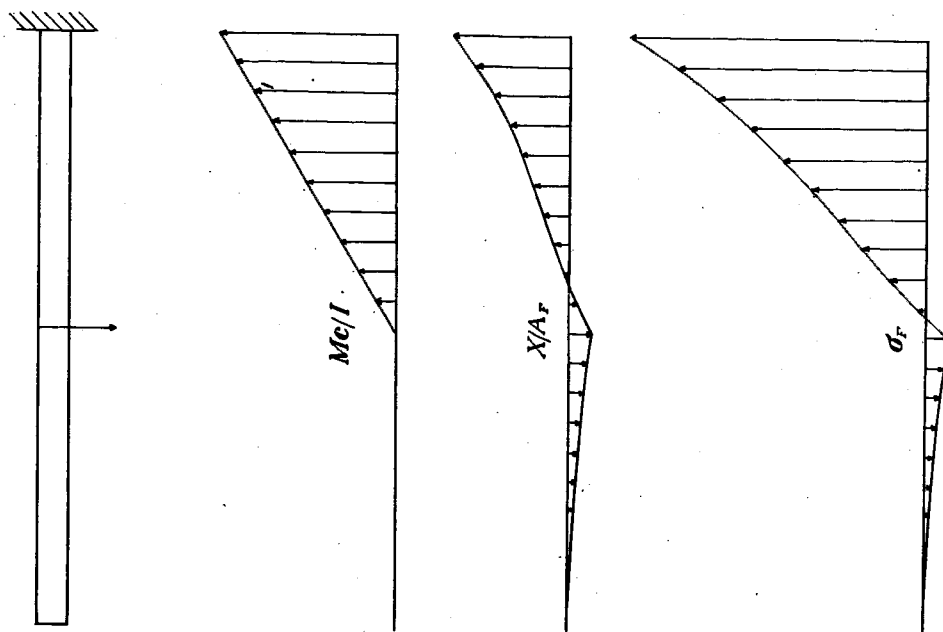Figure 2.- Convention for numbering bays.

Figure 5.- Box beam with cambered covers.

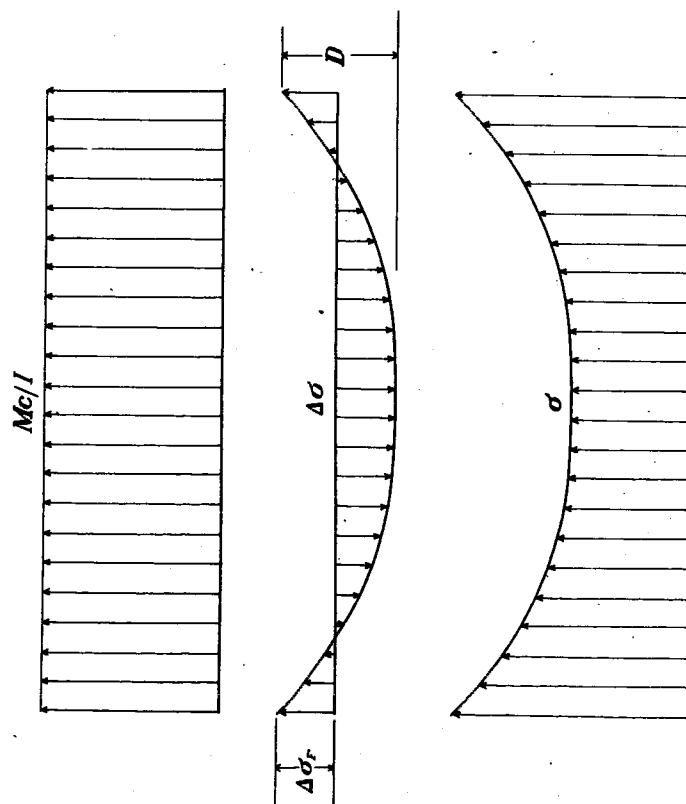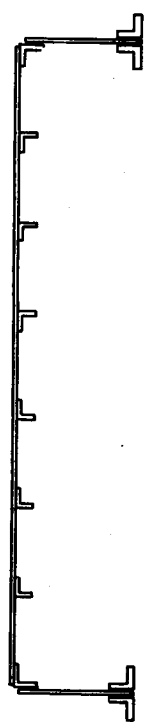Figure 4.- Flange stresses caused by concentrated load.

Figure 3.- Chordwise distribution of stringer stresses.